Exercise 51

Let $B(t)$ be the number of US \$20 bills in circulation at time t. The table gives values of this function from 1990 to 2010, as of December 31, in billions. Interpret and estimate the value of $B'(2000)$.

t	1990	1995	2000	2005	2010
$B(t)$	3.45	4.21	4.93	5.77	6.53

Solution

$B'(t)$ represents the rate that the number of US \$20 bills increases as t increases. Its units are billions per year. To determine the best estimate for $B'(t)$ at $t = 2000$, calculate the slopes of the secant lines nearest to this time,

$$m_1 = \frac{P(2000) - P(1995)}{2000 - 1995} = \frac{4.93 - 4.21}{5} = 0.144$$

$$m_2 = \frac{P(2005) - P(2000)}{2005 - 2000} = \frac{5.77 - 4.93}{5} = 0.168,$$

and take their average.

$$B'(2000) \approx \frac{m_1 + m_2}{2} = 0.156$$

This means the number of US \$20 bills is increasing by 156,000,000 per year on December 31, 2000.